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Exploring Aspects of Mathematical Visualization of Junior High School Student in a Problem-Solving Task

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ABSTRACT

The aim of this research was to explore the mathematical visualization process of Junior High School students in solving the task. This is a qualitative research which aims to explore the mathematical visualization process of Junior High School students in solving a problem-solving task. Data were collected from the written work and the interview of 18 female field independent students on mathematical visualization test consisting of contextual problem-solving task. Results showed that in the aspects of generating object, the students represented the object in the shape of three dimensions because they paid attention to the object from the side position. In the aspect of inspecting object, the students employed partition-object strategy to calculate. To make it easier to solve, the students scanned the objects in the form of two dimensions and three dimensions. Results of studying all the aspects of visualisation also suggest that mathematical visualization process does not occur hierarchically.

Keywords: Mathematical Visualization, Problem-Solving, Junior High School.

1. Introduction

Solving mathematical problem could not be separated from how individuals process their cognition. The process related with the way how they receive information, how they process information and decide the result. However, the fact showed many students learn to solve mathematical problem only by memorizing procedures without understanding the mean behind the procedures. According to Presmeg (1986), in the process in solving mathematics task, students tend to memorize the procedure and mathematical procedure and they used numbers and terms which become key words.

In solving mathematics task, visualization is one of essential aspects involved in the process of deciding the result. It is relevant with the statement of Utomo et al. (2017) stating that visualization has an essential role in developing thought, understanding systematically, and in transition from thinking concretely to abstractly relevant in solving mathematics task. Visualization used in solving the task could be a powerful tool in exploring mathematics task and gave a meaning for the mathematical concepts and their relation Rösken and Rolka (2006). Some research discuss about the benefit of employing visualization, one of which is related to graphic model in mathematics Rösken and Rolka (2006); Lavy (2006). However, there are limited studies nowadays which explores visualization in mathematics education mainly for solving contextual problems.

Hershkowitz (2015) defines visualization as cognitive processes or actions where the individuals improved the connection between an internal construct and something accessed through senses. A connection could be made in two ways. An action of visualization consisted of mental construction from the object or process, where an individual would associate with the object or the event achieved by them externally. A visualization activity can be identified both internally and externally as a construction or process in thought. Visualization is also defined as the process of transforming information in the perception form so that the result presented could connect existing data Presmeg (1986).

Visualization process tend to emphasize how to use geometric illustration which is related to mathematics concept. Besides, mathematical visualization process is more useful in learning mathematic, especially in problem solving Utomo et al. (2017). Mathematical visualization is done by individual student in creating relation of thought and accesible thing by sense as image creating process Lavy (2006). In this research, visualization was a mental in manipulating, exploring, and recognizing mathematical idea as the creativity and interpretation in mind through the aspects of image generation, image in-

spection, image scanning, and image transformation. Morover, mathematical visualization is also defined as mental activity or an individual cognitive activity in exploring and manipulating as creativity during finishing the problem with the aim of communicating information, thought and developed the idea known before obtaining higher understanding Utomo et al. (2017)

This research focus on the mahematical visualization as a mental activity in developing information and idea by using process of image generation, image inspection, image scanning, and image transformation in solving contextual problem. Image generation which is the ability to form visual image, required activating stored visual information and used it to create a pattern in a spatial short-term memory structure. Image inspection is the ability to interpret a pattern in the visual buffer as depicting an object or part of the object. Image scanning is the systematic shifting of attention over an imaged, played a critical role in this ability. Image transformation is the ability to rotate or otherwise altered an imaged pattern. The explanation for each aspect of mathematical visualization in solving the contextual task would be presented in Table 1 as the following:

Table 1: Mathematical Visualization Aspect.

Mathematical Visualization Aspect	Indicators of Mathematical Visualization Aspect	Description
Image generation	Exploring the information from the data as the representation	The subject identify and present the items in the visual forms, algebra symbols which were used to solve the task
	Recognizing the object related to the background knowledge	The subject connects visual ideas with the experience and involved the background knowledge.
Image inspection	Investigating the object to clarify the ideas	The subject pays attention to every item and expressed more specifically for each object given.
	Determining the strategy as the possible alternatives.	The subject determines the strategy as the solution suitable with the visual idea

Mathematical Visualization Aspect	Indicators of Mathematical Visualization Aspect	Description
Image scanning	Copying the object as the possible alternatives	The subject represents some parts or all of the objects and simplify them based on the idea to ease the alternative possibility.
Image transformation	Manipulating the object for the alternative solution	The subject changes the object through (rotation, translation, reflection, and dilatation) as possible alternatives in solving the task

In the theory of Piaget cognitive development, he stated that there are four stages of different ages and changed in the time of childrens cognitive development, namely sensory motor stage (0-2 years old). In this stage, the children involved in their surroundings but they were not able to create internal picture related to the environment. At the stage of preoperational (2-7 years old), the children developed basic concepts such as: time, space, mass, cause and effect, and continued by using symbols. The concrete operational stage was in (7-11 years old). In this stage, the children had an order thought more organized, less selfish, however there was a limit in abstract approach or imaginative one. At the formal operation stage (more than 12 years old), children mastered complex mental operation and related to the abstract and concrete concepts. Clearly, Junior High School students were in the stage of formal operation. Meanwhile, according to Makina and Wessels (2009) visualization is a central component from many processes to make transition from the mind from the concrete to the abstract ones.

In solving the task, each subject had different characteristics. Psychology is one of the studies which mostly identify and indicate individual differences and affect learning process, such as intelligence, skill, cognitive style, learning style, and background knowledge. Zimmermann and Cunningham (1991) stated that by identifying the cognitive style of the subject, teacher could be helped to understand how someone organized and represented information.

Related to the representation of the information, naturally, it could not be separated how the subject expressed in the form of picture and mathematical expression. Cognitive style also had an essential role in the process

of mathematics visualization, mainly when how the information received until the use of information to solve the task Zimmermann and Cunningham (1991) and Utomo et al. (2017). Psychologically, cognitive style is divided into two categories, namely field independent and field dependent. Field independent cognitive style is an individual characteristic that tends to think analytically, reorganizes learning materials according to self-interest, formulates their own learning goals internally so that they can learn step by step, responds well to words, and require visual reinforcement.

Meanwhile, field dependent cognitive style is an individual characteristics who tend to think globally, follow the structure of learning materials as they are, and follow the existing learning objectives. Furthermore, Within in Utomo et al. (2017) states that a field independent individual prefer to be intrinsic, structured and organized movement in learning, as well as independent in the skills of restructuring theory and critical of selecting situational stimulation when compared with field dependent.

Based on the reason, the researchers were encouraged to explore mathematical visualization process of Junior High School students who had cognitive style field-independent as the case study when they solved the mathematics questions.

2. Method

The design of this research is descriptive explorative with qualitative approach. The data collection and data analysis follows Creswell (2017). Participants in this study consisted of 32 students of a Junior High School at Jombang city, Indonesia. To obtain the interviewee participants, as many as 24 female and 8 male students of grade eight were examined with a Group Embeded Figures Test (GEFT) adopted from Witkin to classify the type of each students cognitive style. The result of this test indicates that a total of 18 female students are field independent and 6 female students are field dependent, while 5 male students are field independent and 3 male students are field dependent. In this study case study, the authors only concern on independent field female students to explore aspects of visualization Ferrini-Mundy. (1987). Thus, such 18 field independent female students resulted from GEFT were given the following mathematical visualization test.

A swimming pool with length 50 meters and width 25 meters which the depth was differentiated into two categories, namely adult and teenager category. The adult category was 3 meters and the teenager category was in depth

in 30 meters and teenager category was in depth in 1.5 meters with the length of 17 meters, while the basic border of the two categories was made in slope. How much liters of water needed to fulfill the swimming pool? Explain your answer!

Based on the students responses on the abovementioned problem, the researchers analyzed those responses with the aspects of mathematical visualization. The relationship between the indicators of visualization and the students written work ressponses would be presented in Table 2 as the following.

Table 2: Aspects of mathematical visualization by field independent female students.

Indicators of Mathematical Visualization Aspects	Participants																	
	SF	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Exploring the information from the data as the representation	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Recognizing the object related with the background knowledge	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Investigating the object to clarify the ideas	x	x	x	x		x		x	x			x	x				x	x	x
Determining the strategy as the possible alternatives.	x	x		x				x					x				x		x
Copying the object as the possible alternatives	x	x		x				x	x				x				x	x	x
Manipulating the object for the alternative solution	x		x		x				x			x						x	

x indicates that the corresponding students meet the indicators of mathematical visualization

Table 2 shows that only one student who meet all the indicators of mathematical visualization, while the other students were foud to have some inadequacies of one/some visualization indicators. It underlies the researchers to select SF1 as the participants of this study. Furthermore, the researchers conducted a semi-structured interviews based on SF1s written responses on the problem to further explore every aspect of mathematical visualization.

Credibility of the data used time triangulation. The data analysis referred to the mathematical visualization aspect through the steps namely data categorizing data, reducing data, presenting the data, and making conclusion related to the mathematical visualization aspect in the process of solving the task.

3. Findings and Discussion

Results drawing on the mathematical visualization processes of the 18 female students with field independent cognitive style referred to the mathematical visualization aspects are summarized in Table 3.

Table 3: Findings in Mathematical Visualization Process.

Mathematical Visualization Aspects	Indicator of Mathematical Visualization Aspect	n	p(%)	Findings when the students did the contextual task
Image generation	Exploring information from the data as the representation	18	83,30%	The subject represented the swimming pool in the forms of two dimension and three dimension, such as rectangle, triangular, trapezoidal, and cuboid. The students seemed to give the sign to the size for each item, such as bracket, circle and arrow.
	Recognizing the object related with the background of knowledge			The subject identified the name of the object based on the characters of geometry characteristics.

Mathematical Visualization Aspects	Indicator of Mathematical Visualization Aspect	n	m(%)	Findings when the students did the contextual task
Image inspection	Investigating the object to clarify the ideas	13	70,80%	The process in choosing the way to solve the task, some students did *object partition*, namely making some planes which represented each part such as adult category and teenager category and then continued with determining each volume. Meanwhile, *general object* was the subject determined all the object volume and reduced each part.
	Determining the strategy as the possible alternatives			
Image scanning	Copying the object as the possible alternatives	9	25%	Some students made some planes which were parts of the swimming pool. Some planes made were: triangular, trapezoidal, cuboids, triangular prism.
Image transformation	Manipulating the object for the alternative solution	6	15,10%	Some students did rotation to the object made both in horizontal and vertical.

*participants : 18 female students of field-independent
*n : the number of the students reached the aspects
*p : the number of subject percentage n reaching each indicator

Based on the findings above, in the aspect of generating object, most of student start to identify the given information, the questioned elements and information presented in the form of picture. It can be seen from 83,3% students who activate the indicator of generating object. In the aspect of inspection object, 70.8% of the students observed and determined the strategy used. However, few students (25%) scanned the object to ease the finishing stage. Some of students tend to imagine, which made it become more difficult to determine the result. Finally, at the transforming object stage, there were only few students found to reach this stage. This is because the students tend to imagine without representing into the answer sheet.

In particular, the case study from the finding of the research referred to the aspect of mathematical visualization process in solving mathematics questions as the following:

3.1 Aspect of generating object

Aspect of generating object focused on the way how the students explored and recognized the information from the question. The finding showed that the students identified the information such as size and representation of swimming pool. The students represented it in 3-dimension with the size. The representation of the swimming pool was conducted by field-independent students shown in Figure 1 as the following.

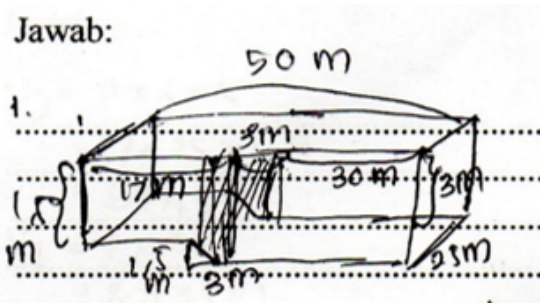


Figure 1: Representation of the swimming pool by the students.

Based on Figure 1 above, the subject showed the representation of the swimming pool with the length of 50 meters, and the width of 25 meters. Besides, the subject also showed each size category such as the length of adult category was 30 meters with the depth was 3 meters, and the length of teenager

category was 17 meters with the depth was 1.5 meters. The following interview was between the researcher and a subject.

Pi	:	(i-th) statement by the researcher
SPFIi	:	(i-th) answer by the subject

P1	:	Explain the picture which you have made, please (Figure 1)!
SPfi1	:	I made a picture of swimming pool divided into three parts, namely adult category, teenager category, and the delimiter from the two categories.
P2	:	Why was the picture like this?
SPfi2	:	To make it easier to see
P3	:	What did you mean?
SPfi3	:	It was easy if this plane (Figure 1) were geometry
P4	:	From what side did you see it?
SPfi4	:	I saw it from the top side

In the contextual case, the subject developed the imaginative information based on the situation which ever happened. The subject related the information in the form of question to the background knowledge such as the characteristic of geometry.

The subject identified the delimiter area from the two categories by giving hatching sign. To make easier to understand the picture made, the subject showed the perspective of the swimming pool from the side position and top position. In other words, the subject showed the representation made was the plane of 3-dimensions.

3.2 Aspect of inspecting object

In this aspect, it focused on how the subject inspected an object to clarify the idea and determined the strategy as the alternative solution. The subject observed each part of the representation of the swimming pool based on the characteristic such as a part of swimming pool for adult category, a part of swimming pool for teenager category, and a part of swimming pool for the delimiter from the two categories.

Based on Figure 1, the subject assumed that the part of the swimming pool for adult and teenager categories were cuboids, because each item had different size. This following interview was between the researcher and the subject.

P5 : From this picture (Figure 1), what shape is it?
SPfi5 : The adult and teenager swimming pools are cuboids, but the delimiter is a prism with the trapezoidal base.
P6 : Why is the shape of adult and teenager swimming pool considered as cuboids?
SPfi6 : Because this plane has different size of measurement, namely 30 meters, 3 meters and 25 meters, while the swimming pool for the teenager category had 17 meters, 25 meters, and 1.5 meters
P7 : Is there another reason?
SPfi7 : Yes, because the sides face each other
P8 : Then how is the delimiter?
SPfi8 : The delimiter is in the form of prism with trapezoidal base.
P9 : Why is it trapezoidal?
SPfi9 : Because the swimming pool base is slope or tilted, but it could only be seen from the inside of the swimming pool, because it could not be seen from the outside

The effect of observing the swimming pool as a part or for each item, the subject determined the volume of the swimming pool with the object partition strategy. This strategy focused on how the way to find solution by observing each part then accumulated it. This following interview is between the researcher and the subject.

P10 : Explain how do you calculate this volume (Figure 1)?
SPfi10 : Firstly, I investigated the volume of the swimming pool for adult category with the cuboids pattern (shown in Figure 2 as the following),

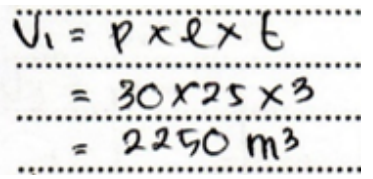


Figure 2: The volume of the swimming pool for adult category.

then continued by looking for the volume of the swimming pool for teenager category with the cuboids pattern (shown in Figure 3 as the following)

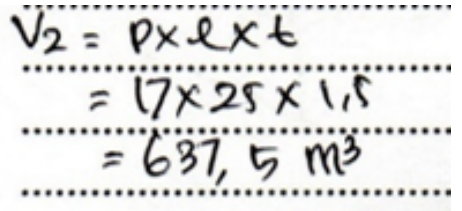


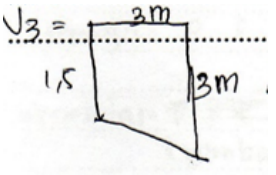
Figure 3: The volume of the swimming pool for teenager category.

P11 : Why did you use the way?
SPfi11 : To make it easy to do it, because if it was done thoroughly, it would be difficult.

3.3 Aspect of scanning the object

When the subject determined the volume of the delimiter from the two categories of the swimming pool, the subject scanned the object in another place. It aimed to ease in understanding the delimiter representation. The representation scanned the object shown in the following Table 4.

Table 4: The Representation Scanned the Object.

Code	Object Representation	Explanation
SO1		From Figure 1, the subject made the object representing the delimiter area in the form of 3-dimensions, namely prism shown in SO1
SO2		The subject showed the object in two dimension, where the object is the part of SO1. It is right angled trapezium shown by SO2

*SOi is ith was the representation process of the delimiter object in the aspect of object scanning.

3.4 Aspect in transforming object

This aspect more focused on how the subject did manipulation to the object as the possible alternative solution. The subject did object transformation when the subject determined the volume of delimiter of the two swimming pool categories. The following interview was between the researcher and the subject.

P12	:	How did you find out the delimiter volume?
SPfi12	:	I used prism pattern with trapezium base
P13	:	Why did you use the pattern?
SPfi13	:	To make it easy to count.
P14	:	How could you do it?
SPfi14	:	I rotated the plane so that the trapezium became the base (SO1), then I counted the area of (SO2), then I multiplied it with the height of prism. Shown in figure 4 as the following.

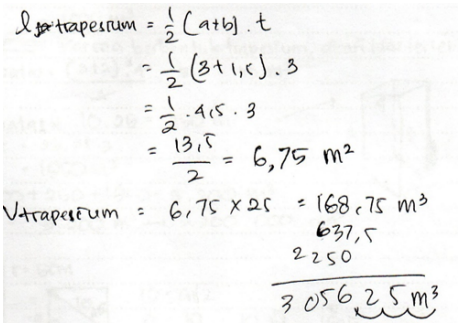


Figure 4: The volume of delimiter of the two swimming pool categories.

The research finding showed that the mathematical visualization process with the subject of field-independent cognitive style in solving contextual task involved experience to represent the object. The subject imagined the position before representing the object. Some of the students developed the information in the question to make data sufficient. Mathematical visualization process consist of four aspects: generating object, inspecting object, scanning object, and transforming object. Findings presented in this part shows that mathematical visualization process did not occur hierarchically. Instead, it is about

how individuals connect each aspect of visualization. Referring to the background of the students in this study, the students has yet to show a higher understanding of solving problem-solving task considering their mathematical visualization. According to Zimmermann and Cunningham (1991) "Visualization is the capability, process and product of creation, interpretation, use and reflection of images, drawings, diagrams in our minds, on paper or by means of technology, for the purpose of describing and communicating information, developing previously unknown ideas and promoting understanding".

While on the aspect of object transformation, the subject is already higher, because the manipulation of objects is done internally. In other words, students for Learning Science make a change in the mind. Jones (2001) reveals that visualization is one of the skills in thinking related to imagining things and rotating in a way from various directions either as two dimensional or three dimensional. This is in line with the results of research that shows that each subject performs a circular motion, shifting and developing the object. Skills using external and internal objects. Matters relating to the use of information and measurements used to solve prior and subsequent problems can solve contextual problems, since students have points of view from different directions. As well as internal internal play, where people rotate objects without movement on movement and movement.

The existence of different strategies used by field independent female students, namely the object partitioning strategy and general object strategy, shows the relationship between visualization and creative thinking. However, most field independent female students used more partitioning strategy, because it is considered easier to understand each part of the object than the object as a whole. It shows that there is a link between visualization and creativity in solving mathematics problems. It is in line with the opinion of Utomo et al. (2017) that both visualization ability and creative thinking are important components in problem solving.

In addition, according to visualization can enhance one creativity through reflection of existing ideas and reflections on objects of various dimensions. An interesting finding is what are called objects that are referred to as objects that fit the mathematical rules. It shows that people are in school. In addition, this research can be used as a reference for education in Indonesia to collect material from one of the lessons at a junior high school. Besides, the aspect combination done by the female field independent was to ease the understanding in answering the question. Related to Phillips et al. (2010) "in visualization, pictures combine aspect of naturalistic representation with more formal shape to enhance cognitive understanding". The subject scanned the

object with the aim to simplify the object in solving the task. In transforming the object, the subject manipulated the object by rotation to determine the delimiter volume.

4. Conclusion

This study tries to explore the mathematical visualization process Junior High School students in solving mathematics task from four aspects. Generating object was carried out by the subject when she wanted to explore and to recognize the background knowledge. The subject represented the swimming pool in three dimension. The subject paid attention to the position representing the object, namely in side position.

Choosing the strategy for finding solution was based on the time when the subject observed each area of the object. The subject used strategy of object partition, where the subject counted each area of the object and then accumulated it.

Manipulating object was done when she determined the delimiter volume. It could be seen when the subject did rotation to the delimiter area which was in shape of prism with the base of right angled trapezium. The subject changed the position on the delimiter object because it was rare to find the shape, so the subject manipulated to be able to understand. Thoroughly, in exploring the mathematical visualization process, the subject showed the plot of the process did not happen in hierarchy

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